# H-dibaryon from Full QCD Lattice Simulations

Takashi Inoue, Nihon Univ.

for HAL QCD Collaboration

| | |
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| K. Sasaki | Univ. Tsukuba |


RBRC, HHI workshop, Feb 29, 2012

# Introduction

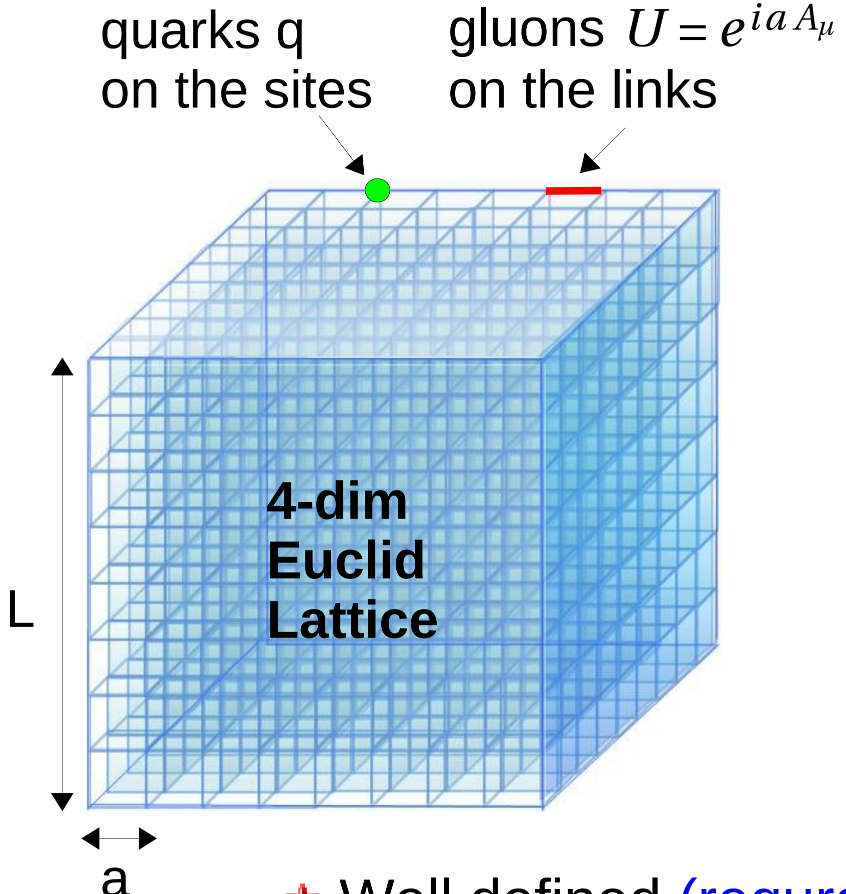
- H-dibaryon: predicted compact 6-quark (B=2) state
  - R. L. Jaffe, Phys. Rev. Lett. 38 (1977) [MIT Bag model]
  - One of most famous candidates of exotic-hadron.
  - based on the observation of no Pauli exclusion due to $1_F$ nature and a large attractive contribution from one-gluon-exchange.
  - Does H exist in nature? $B_H > 7$ MeV is ruled out by $_{\Lambda\Lambda}$He. Possibility of a shallow bound state or a resonance still remains.
- Hyperon interaction (YN, YY int.)
  - are important for phys. of super-nova, neutron star and so on.
  - however, are not well known due to lack of experimental data.
- Our purpose and goal
  1. We reveal BB int., including existence of the H-dibaryon, directly from QCD by using lattice simulation.
  2. We get deeper(or intuitive) understanding of BB int.
  3. People can apply the knowledge to many physics. =GOAL

# Plan of this talk

- Introduction
  - Brief background,  Our purpose and goal
  - Lattice QCD Simulation  – brief Introduction –
  - Multi-hadron system in LQCD – our approach –
- Formulas & Setup
  - NBS w.f.,  Potential,  FAQ
  - Lattice,  Action and Facility,
  - Five ensembles,  Why $SU(3)_F$ limit ?
- Results
  - BB interations,  H-dibaryon
  - Hyperon interactions,  H in the real world
- Summary & Outlook

# Lattice QCD Simulation

$$L = -\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a + \bar{q}\gamma^\mu \left( i\partial_\mu - g t^a A^a_\mu \right) q - m\bar{q}q$$




Vacuum expectation value

$$\langle O(\bar{q}, q, U) \rangle$$

path integral

$$= \int dU\, d\bar{q}\, dq\, e^{-S(\bar{q},q,U)}\, O(\bar{q}, q, U)$$

$$= \int dU \det D(U) e^{-S_U(U)}\, O(D^{-1}(U))$$

quark propagator

$$= \lim_{N\to\infty} \frac{1}{N} \sum_{i=1}^{N} O(D^{-1}(U_i))$$

{ $U_i$ } : ensemble of gauge conf. U generated w/ probability $\det D(U)\, e^{-S_U(U)}$

- Well defined (regularized)
- Manifest gauge invariance
- Fully non-perturbative
- Highly predictable

# Multi-hadron system in LQCD

- Conventional : use energy eigenstate (eigenvalue)
  - Lüscher's finite volume method for phase-shift
  - Infinite volume extrapolation to get bound state energy
- HAL : utilize a potential $V(r) + \ldots$ from the NBS w.f.
  - ie. an effective theory which reproduce T matrix from QCD
  - Advantages
    - No need to separate E eigenstate.
      Just need to measure the NBS w.f.
      Then, potential can be extracted.
    - Demand a minimal lattice volume.
      No need to extrapolate to $V=\infty$.
    - Can output many observables.

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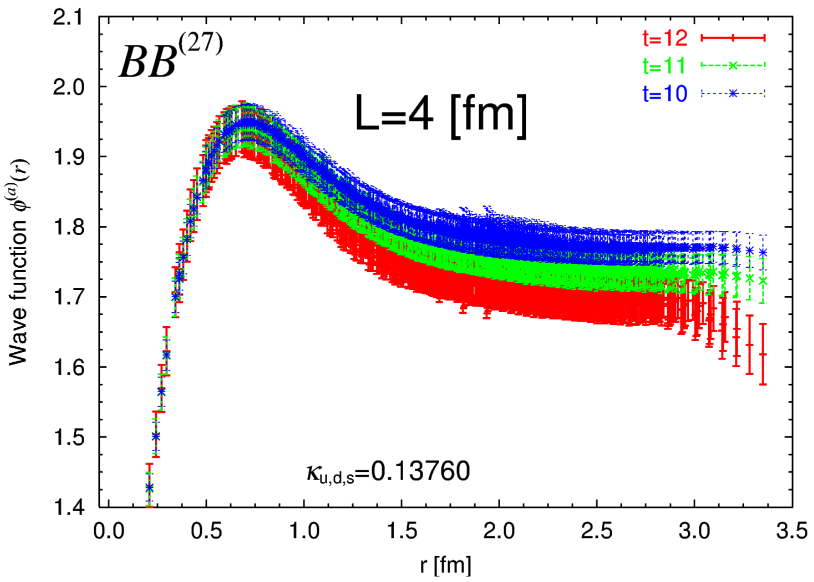

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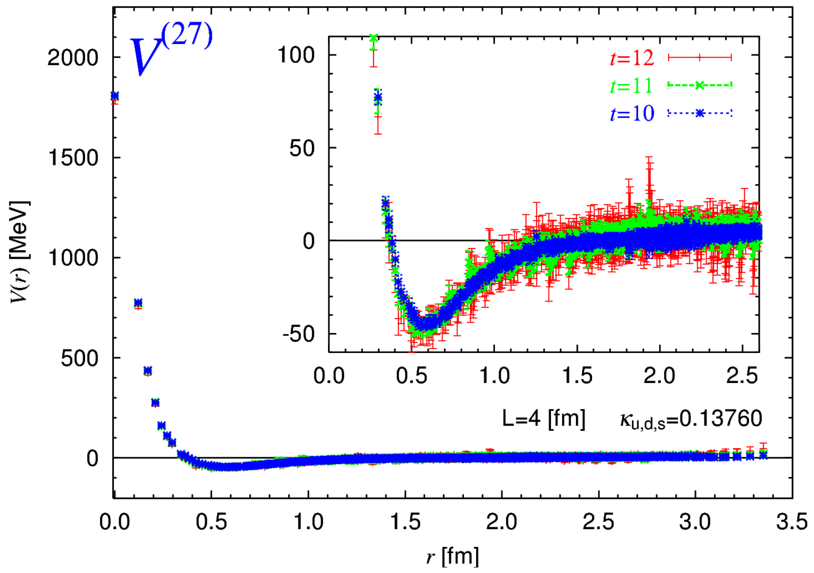

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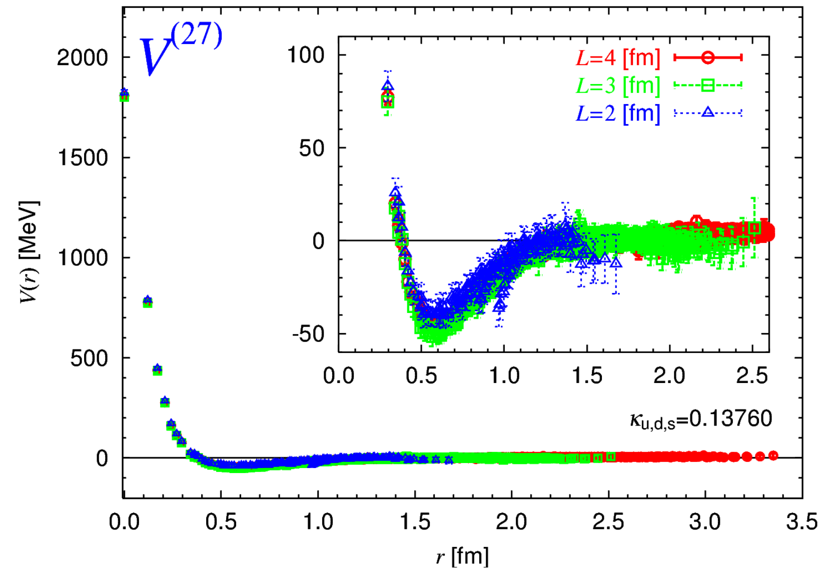

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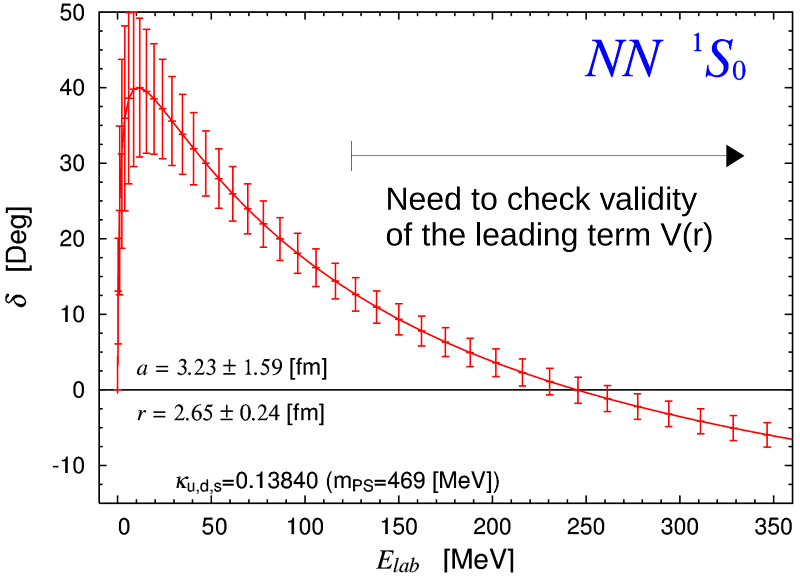

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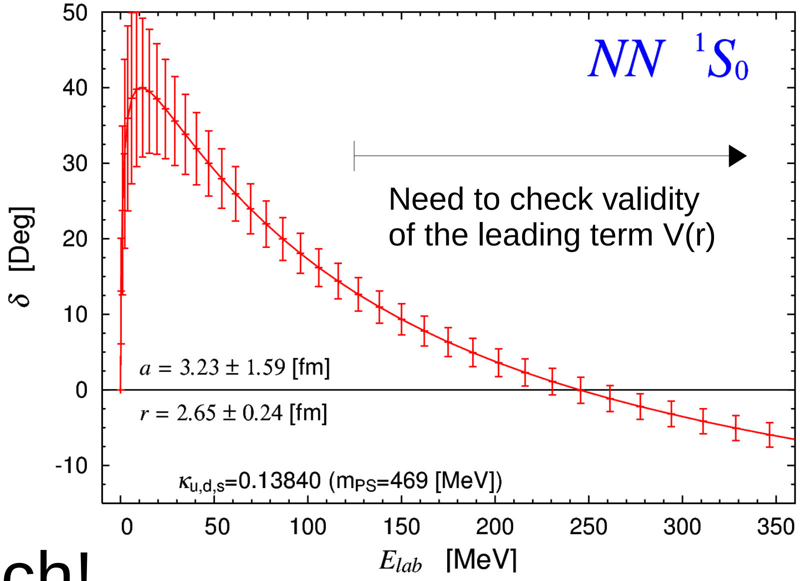



★ We can probe the H in this approach!

as far as H has BB component.

# Formulas and Setup

# Nambu-Bethe-Salpeter w.f.

- NBS wave function

the same

$$\psi^{(a)}(\vec{r}\,,\,t) \stackrel{\text{def}}{=} \sum_{\vec{x}} \langle 0 | B_i(\vec{x}+\vec{r}\,,t) B_j(\vec{x}\,,t) | B=2, \text{a-plet} \rangle$$

QCD generated state

$$\propto \sum_{\vec{x}} G^{(a)}(\vec{x}+\vec{r}\,,\vec{x}\,,\,t)$$

4-point function $G^{(a)}(\vec{x}\,,\vec{y}\,,\,t-t_0) = \langle 0 | B_i(\vec{x}\,,t) B_j(\vec{y}\,,t)\; \overline{BB}^{(a)}(t_0) | 0 \rangle$

sink source

- Point type octet baryon field operator at sink

$$p_\alpha(\underline{x}) = \epsilon_{c_1 c_2 c_3} (C\gamma_5)_{\beta_1\beta_2} \delta_{\beta_3\alpha}\, u(\xi_1) d(\xi_2) u(\xi_3) \quad \text{with} \quad \xi_i = \{c_i,\, \beta_i,\, \underline{x}\}$$

$$\Lambda_\alpha(x) = -\epsilon_{c_1 c_2 c_3} (C\gamma_5)_{\beta_1\beta_2} \delta_{\beta_3\alpha} \sqrt{\frac{1}{6}} \left[ d(\xi_1) s(\xi_2) u(\xi_3) + s(\xi_1) u(\xi_2) d(\xi_3) - 2 u(\xi_1) d(\xi_2) s(\xi_3) \right]$$

- Quark wall type BB source in the flavor irreducible rep.

e.g for flavor-singlet $\quad \overline{BB}^{(1)} = -\sqrt{\frac{1}{8}}\, \overline{\Lambda}\,\overline{\Lambda} + \sqrt{\frac{3}{8}}\, \overline{\Sigma}\,\overline{\Sigma} + \sqrt{\frac{4}{8}}\, \overline{\mathrm{N}}\,\overline{\Xi}$

# Potential

S. Aoki, T. Hatsuda, N. Ishii, Prog. Theo. Phys. 123 89(2010)
N. Ishii etal. [HAL QCD coll.] in preparation

NBS wave function $\psi(\vec{r},t) = \phi_{Gr}(\vec{r}) e^{-E_{Gr} t} + \phi_{1st}(\vec{r}) e^{-E_{1st} t} \cdots$

DEFINE a “potential” through the “Schrödinger eq.” for E-eigen-sates.

$$\left[2M_B - \frac{\nabla^2}{2\mu}\right]\phi_{Gr}(\vec{r}) e^{-E_{Gr} t} + \int d^3\vec{r}\,' U(\vec{r},\vec{r}\,')\phi_{Gr}(\vec{r}\,') e^{-E_{Gr} t} = E_{Gr}\phi_{Gr}(\vec{r}) e^{-E_{Gr} t}$$

$$\left[2M_B - \frac{\nabla^2}{2\mu}\right]\phi_{1st}(\vec{r}) e^{-E_{1st} t} + \int d^3\vec{r}\,' U(\vec{r},\vec{r}\,')\phi_{1st}(\vec{r}\,') e^{-E_{1st} t} = E_{1st}\phi_{1st}(\vec{r}) e^{-E_{1st} t}$$

Non-local but energy independent

By adding equations
$$\left[2M_B - \frac{\nabla^2}{2\mu}\right]\psi(\vec{r},t) + \int d^3\vec{r}\,' U(\vec{r},\vec{r}\,')\psi(\vec{r}\,',t) = -\frac{\partial}{\partial t}\psi(\vec{r},t)$$

$\nabla$ expansion & truncation
$$U(\vec{r},\vec{r}\,') = \delta(\vec{r}-\vec{r}\,')V(\vec{r},\nabla) = \delta(\vec{r}-\vec{r}\,')[V(\vec{r}) + \cancel{\nabla + \nabla^2}\ldots]$$

Therefor, in the leading

$$\boxed{V(\vec{r}) = \frac{1}{2\mu}\frac{\nabla^2\psi(\vec{r},t)}{\psi(\vec{r},t)} - \frac{\frac{\partial}{\partial t}\psi(\vec{r},t)}{\psi(\vec{r},t)} - 2M_B}$$

# FAQ

1. Does your potential depend on the choice of source?

2. Does your potential depend on choice of operator at sink?

3. Does your potential U(r,r') or V(r) depend on energy?

# FAQ

1. Does your potential depend on the choice of source?

➔ No. Some sources may enhance excited states in NBS w.f. However, the contamination is not problem in our new technique.

2. Does your potential depend on choice of operator at sink?

➔ Yes. It can be regarded as the "scheme" to define a potential. Note that a potential itself is not physical observable. We'll obtain unique result for observables irrespective to the choice, as long as the potential U(r,r') is deduced exactly.

3. Does your potential U(r,r') or V(r) depend on energy?

➔ By definition, U(r,r') is non-local but energy independent. While, determination and validity of its leading term V(r) obtained here, depend on energy because of the truncation. However, we know that the dependence in NN case is very small (thanks to our choice of sink operator = point) and negligible at least at $E_{lab.}$ = 0 – 90 MeV. We rely on this in the following. If we find the dependence, we'll determine the next leading term form it.

# Lattice, Action and Facility

| $\beta$ | a [fm] | Sites | L [fm] |
|---|---|---|---|
| 1.83 | 0.121(2) | $32^3 \times 32$ | 3.87 |

- Renormalization group improved Iwasaki gauge and Non-perturbatively O(a) improved Wilson quark
- We thank K.-I. Ishikawa and the PACS-CS group for providing their DDHMC/PHMC code to generate gauge configuration, and the Columbia Physics System for their lattice QCD simulation code.
- We enhance S/N of data by averaging on 4x4=16 source, and forward/backward propagation in time.
- All numerical computation are carried at T2K-Tsukuba.

# Five ensembles

T.I. et.al. Phys. Rev. Lett. 106, 162002(2011)

SU(3)$_F$ limit

| $K_u = K_d = K_s$ | N_cfg | M_P.S. [MeV] | M_Vec [MeV] | M_Bar [MeV] | relative to |
|---|---|---|---|---|---|
| 0.13660 | 420 | 1170.9(7) | 1510.4(0.9) | 2274(2) | New |
| 0.13710 | 360 | 1015.2(6) | 1360.6(1.1) | 2031(2) | add cfg |
| 0.13760 | 480 | 836.8(5) | 1188.9(0.9) | 1749(1) | add cfg |
| 0.13800 | 360 | 672.3(6) | 1027.6(1.0) | 1484(2) | add cfg |
| 0.13840 | 720 | 468.6(7) | 829.2(1.5) | 1161(2) | New |

K: quark hopping parameter

- We've made five ensembles with different value of $K_u=K_d=K_s$ (=quark mass) corresponding to $M_{PS}$ = 1.17 [GeV] to 470 [MeV].
- With lightest quark (bottom row of the table),
  - p.s. meson is a little lighter than the physical kaon.
  - baryon is a little lighter than the physical sigma baryon.
- Now, the simulated hadron world is not so far from the real world, although the SU(3)$_F$ breaking is not taken into account.

# Why $SU(3)_F$ limit ?

- In the limit, convenient basis exist to describe BB int.

$$8 \times 8 = \underbrace{27 + 8s + \boxed{1}}_{\text{Symmetric}} + \underbrace{10^* + 10 + 8a}_{\text{Anti-symmetric}}$$

flavor irreducible rep.

- In S-wave, no off-diagonal interaction exists.

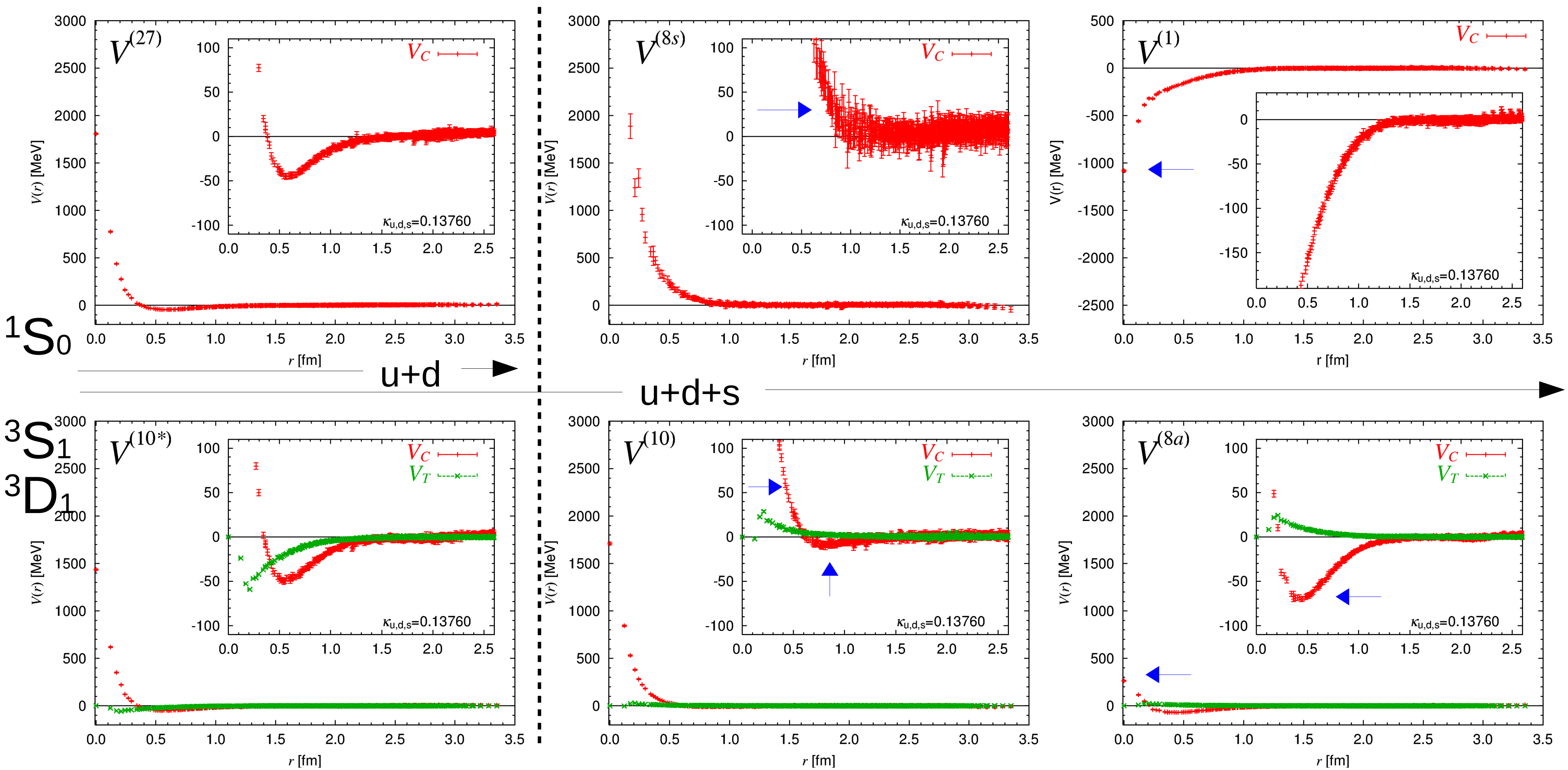
Fermi statistics leads to

$$^1S_0 \;:\; V^{(27)}(r),\quad V^{(8s)}(r),\quad V^{(1)}(r)$$
$$^3S_1 \;:\; V^{(10^*)}(r),\quad V^{(10)}(r),\quad V^{(8a)}(r)$$

- Six $V^{(a)}(r)$ contain essential flavor-spin structure of BB int.
- We can reconstruct all baryon-base interaction (eg. ΛN) by using these $V^{(a)}(r)$ with SU(3) C.G. coefficients.
- The $V^{(a)}$ is useful to pin down physical origin of particular feature, since effective models assume the flavor symm.

# Results

# BB int. in flavor basis




- At the SU(3)F limit corresponding to M $\pi$ = MK = 837 [MeV].
- QM is true at small $r$. Especially, no repulsion in 1F channel.
- This indicate possibility of a bound H-dibaryon in the limit.

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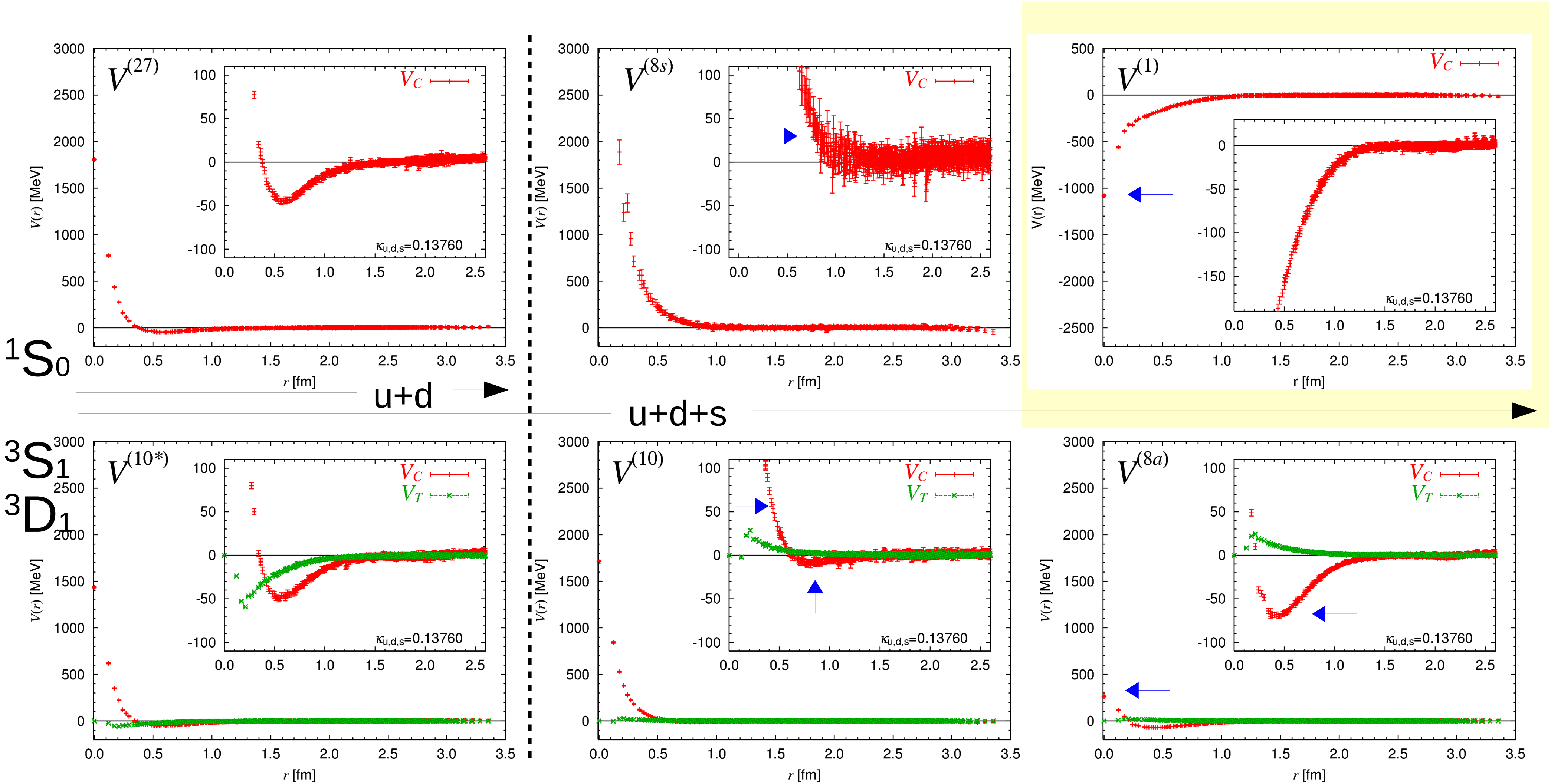


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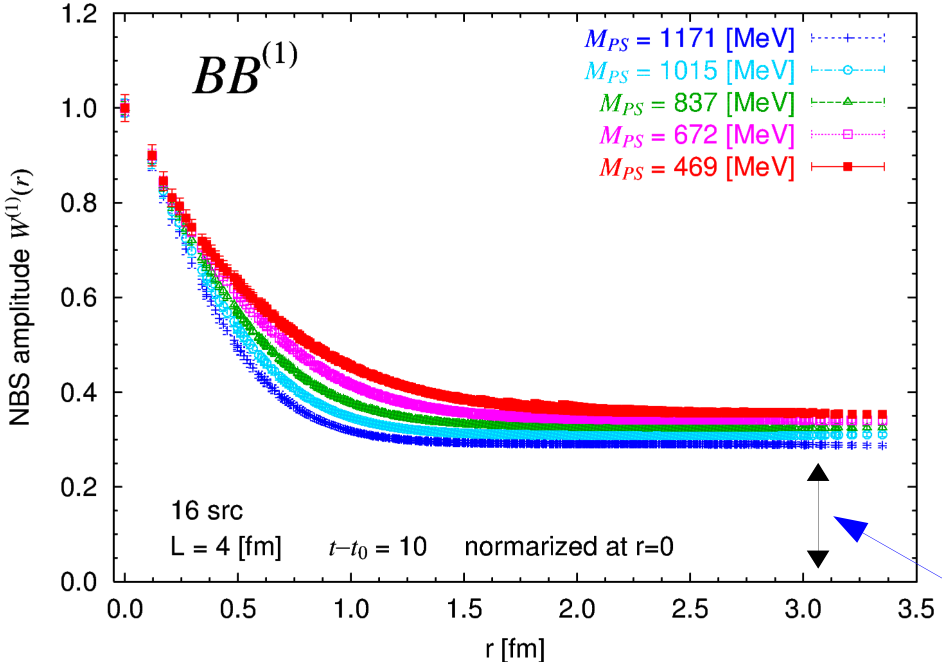
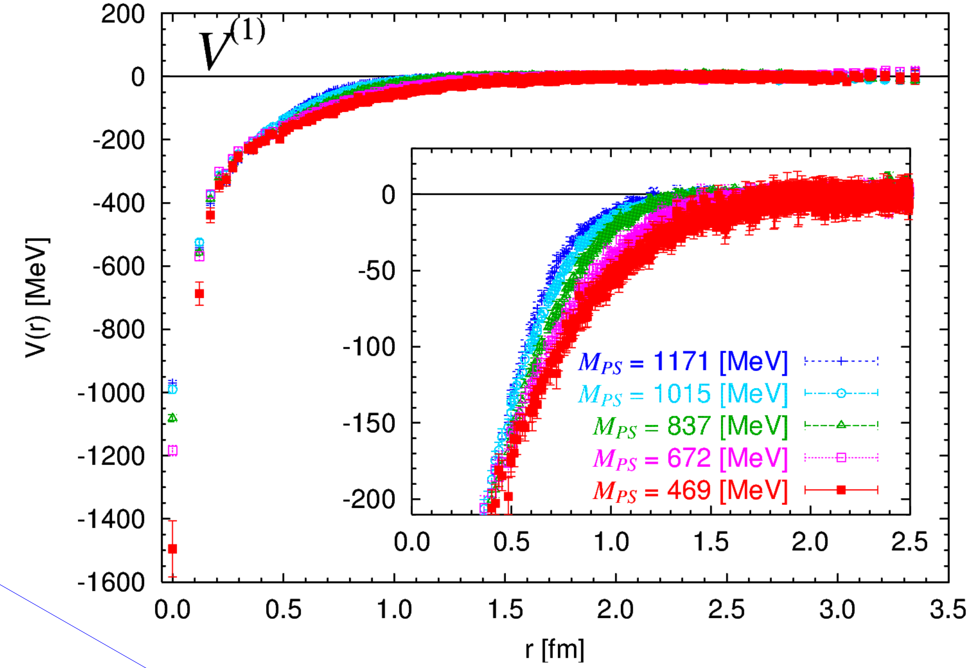
# H-dibaryon

# NBS w.f. & potential

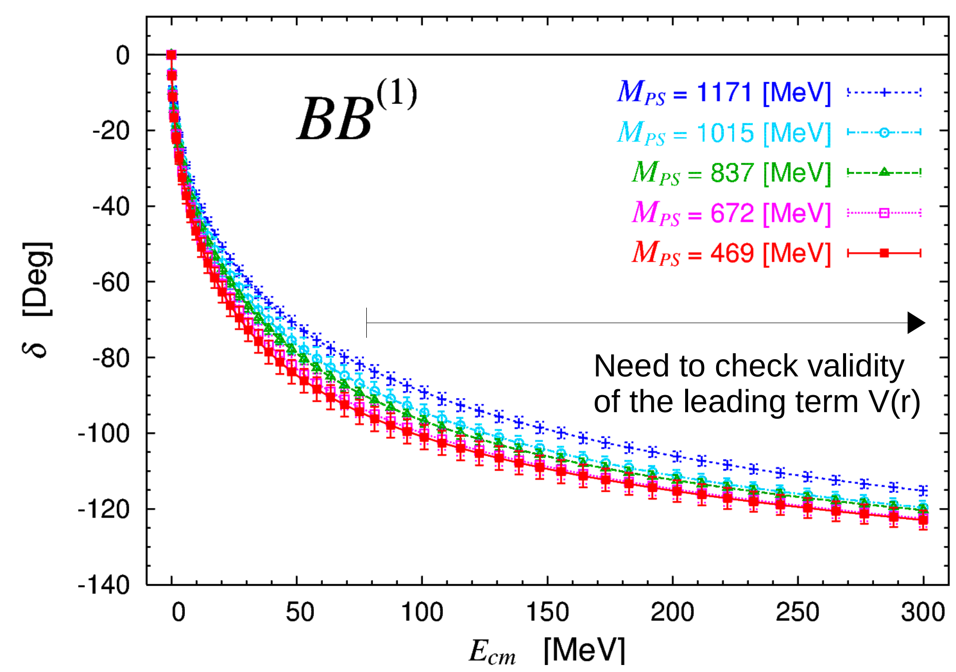


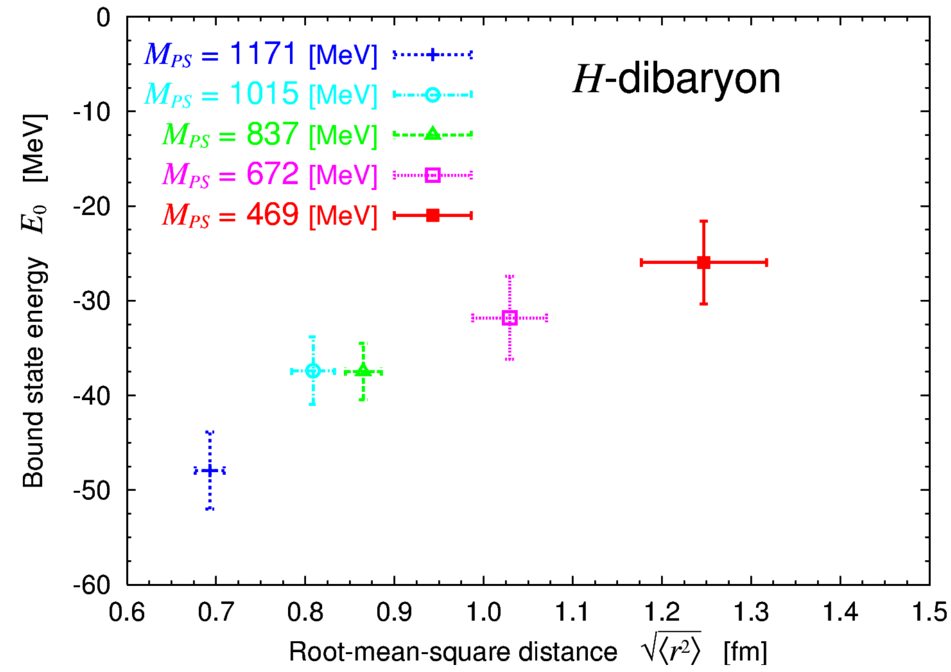




- Left: Measured NBS w.f. of the 1F channel
  - The finite value at large distance is excited states contribution as well as a finite volume effect.
- Right: Extracted potential of the 1F channel
  - $V^{(1)}(r)$ become more attractive as quark mass decrease.

# Observables

T. I. etal [HALQCD collaboration]
Nucl. Phys. A in print, arXive 1112.5926




- Left: scattering phase shift v.s. $E_{\mathrm{cm}}$
  - shows existence of one discrete state below threshold.
- Right: obtained ground state
  - which is 20 - 50 MeV below from free BB ie. 3q-3q.
  - This means that there is a 6-quark bound state in the $1_F$ channel.
  - A stable(bound) H-dibaryon exists in these $SU(3)_F$ limit world!

# Hyperon-Hyperon interaction

# BB int. in baryon-basis

- In flavor SU(3) broken world, e.g. the physical one, the baryon-basis are used instead of the flavor-basis.
- In the SU(3)$_F$ limit, the baryon-base potential $V_{ij}(r)$ can be obtained by a unitary rotation of the potential $V^{(a)}(r)$.

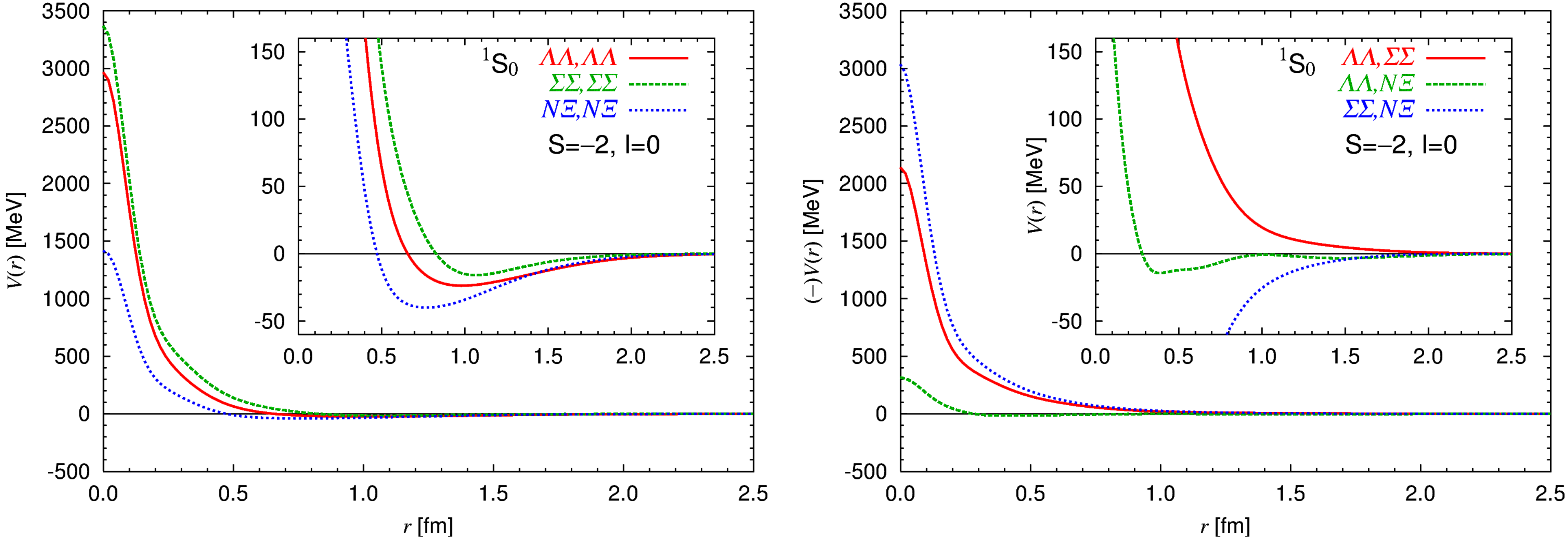
e.g. S=−2, I=0 sector

coupled channel

$$\begin{pmatrix} \langle \Lambda\Lambda | \\ \langle \Sigma\Sigma | \\ \langle \Xi N | \end{pmatrix} = U \begin{pmatrix} \langle 27 | \\ \langle 8 | \\ \langle 1 | \end{pmatrix}, \quad U \begin{pmatrix} V^{(27)} & & \\ & V^{(8)} & \\ & & V^{(1)} \end{pmatrix} U^t = \begin{pmatrix} V^{\Lambda\Lambda} & V^{\Lambda\Lambda}_{\Sigma\Sigma} & V^{\Lambda\Lambda}_{\Xi N} \\ & V^{\Sigma\Sigma} & V^{\Sigma\Sigma}_{\Xi N} \\ & & V^{\Xi N} \end{pmatrix}$$

- I show you potentials $V_{ij}(r)$ at the lightest quark mass ($K_{uds}$ = 0.13840, $M_{ps}$=469 MeV) obtained with the $V^{(a)}$ in an analytic function fitted to data.

# HHI in S=−2, I=0 sector




- ΛΛ – NΞ – ΣΣ coupled. Left: diagonal. Right: Off-diagonal.
- This sector is flavor symmetric (spin singlet), and involves 1F.
- Channel coupling interactions are comparable to diagonal ones, except for small ΛΛ – NΞ transition (sign change must be artifact).
- Interaction is most attractive in NΞ channel, although it has not much meaning because channel coupling is strong.

# H in the real world

## – a trial calculation –

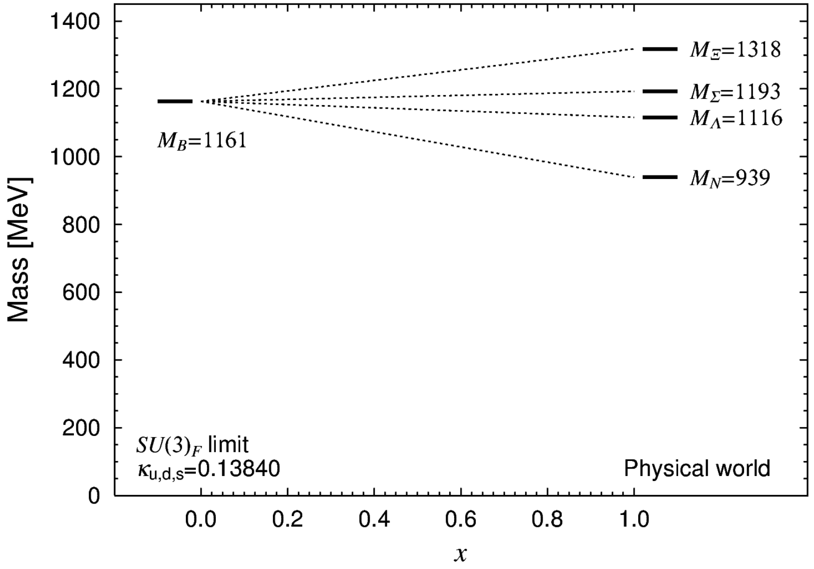
# $J^P = 0^+$ states in S=−2, I=0

- We study scattering in ΛΛ – NΞ – ΣΣ coupled $^1S_0$ channel.

$$T^{\alpha\beta} = V^{\alpha\beta} + \sum_{\gamma} V^{\alpha\gamma} G_{\gamma}^{(0)} T^{\gamma\beta}, \quad G_{\gamma}^{(0)} = \frac{1}{E - H_{\gamma}^{(0)} + i\epsilon}$$

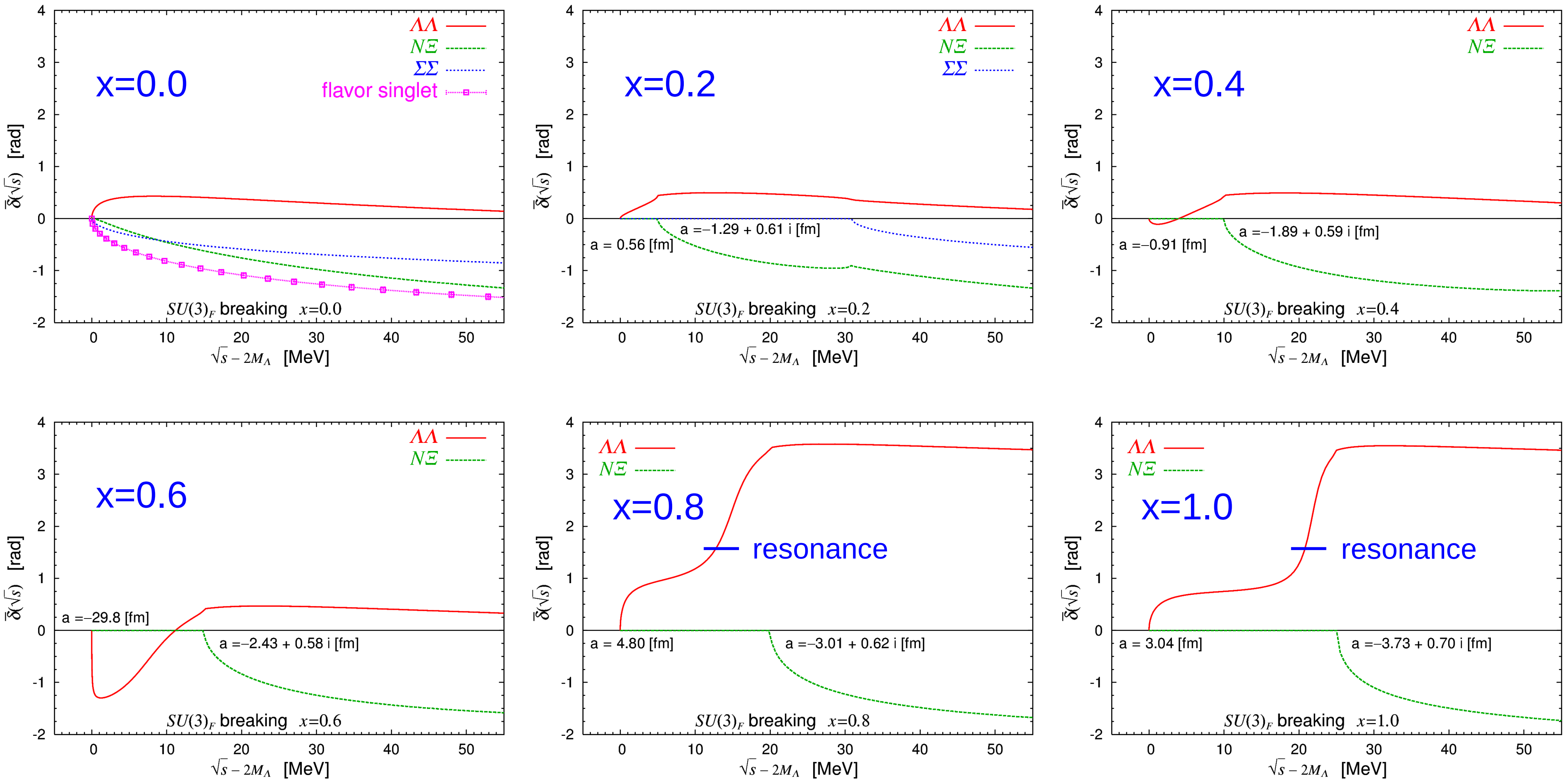
- For baryon masses, we use values interpolated between $SU(3)_F$ limit one at K=0.13840 ($M_{ps}$ = 469 MeV) and physical ones linearly.

$$M_Y(x) = (1-x) M_B^{SU(3)} + x M_Y^{Phys}$$




- For $V^{\alpha\beta}$, we use LQCD results given in the previous slide.
- This is just a trial study or demonstration for the moment!
(based on the assumptions 1. the mass of baryon has major effect,
2. qualitative features of $V^{\alpha\beta}$ remain intact w/ $SU(3)_F$ breaking).

# Phase-shifts

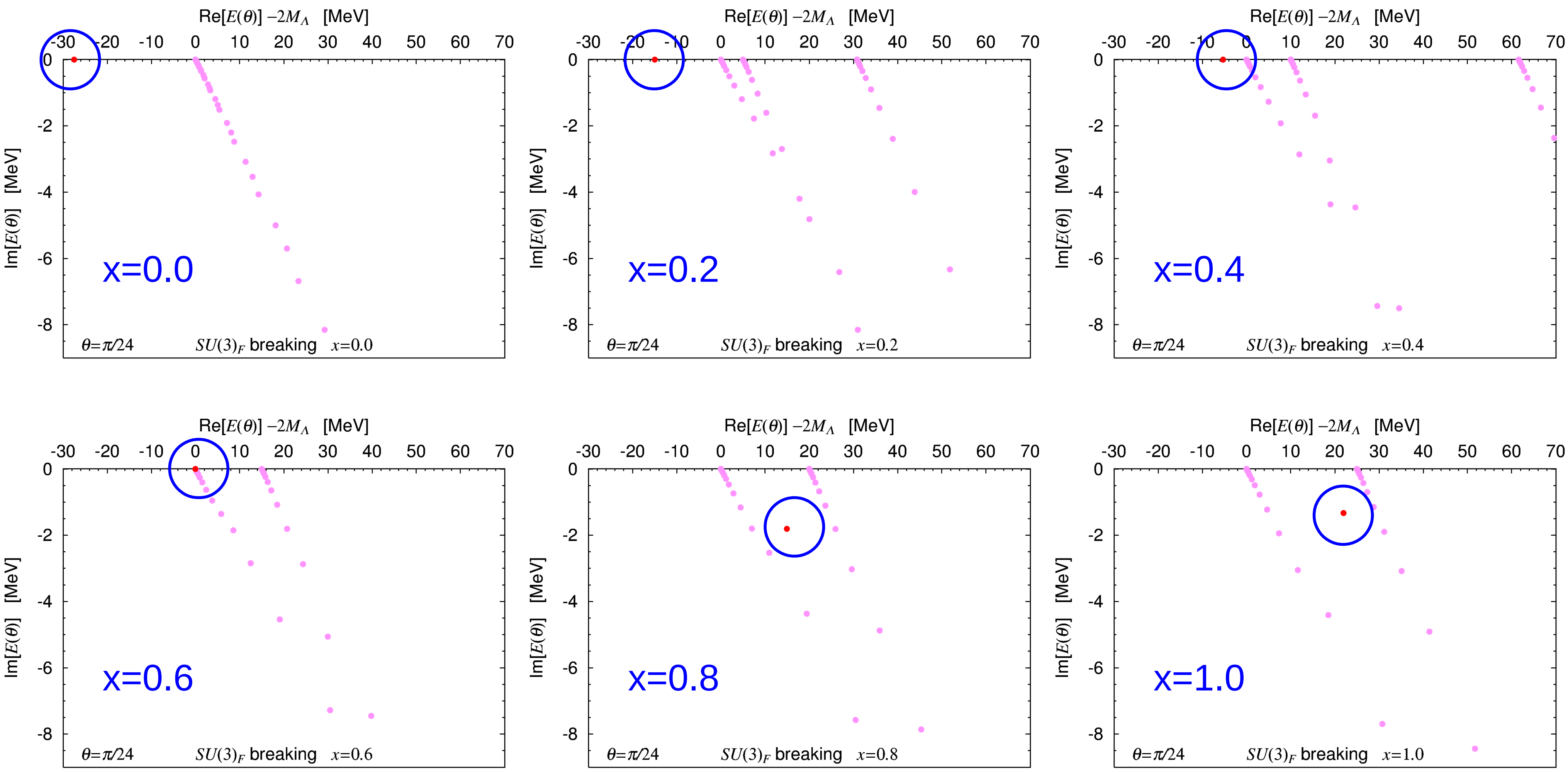



- In the Stapp parametrization, the bar-phase-shifts are defined as

$$S_{ii}^{l=0} = \eta_i e^{2i\bar{\delta}_i}$$

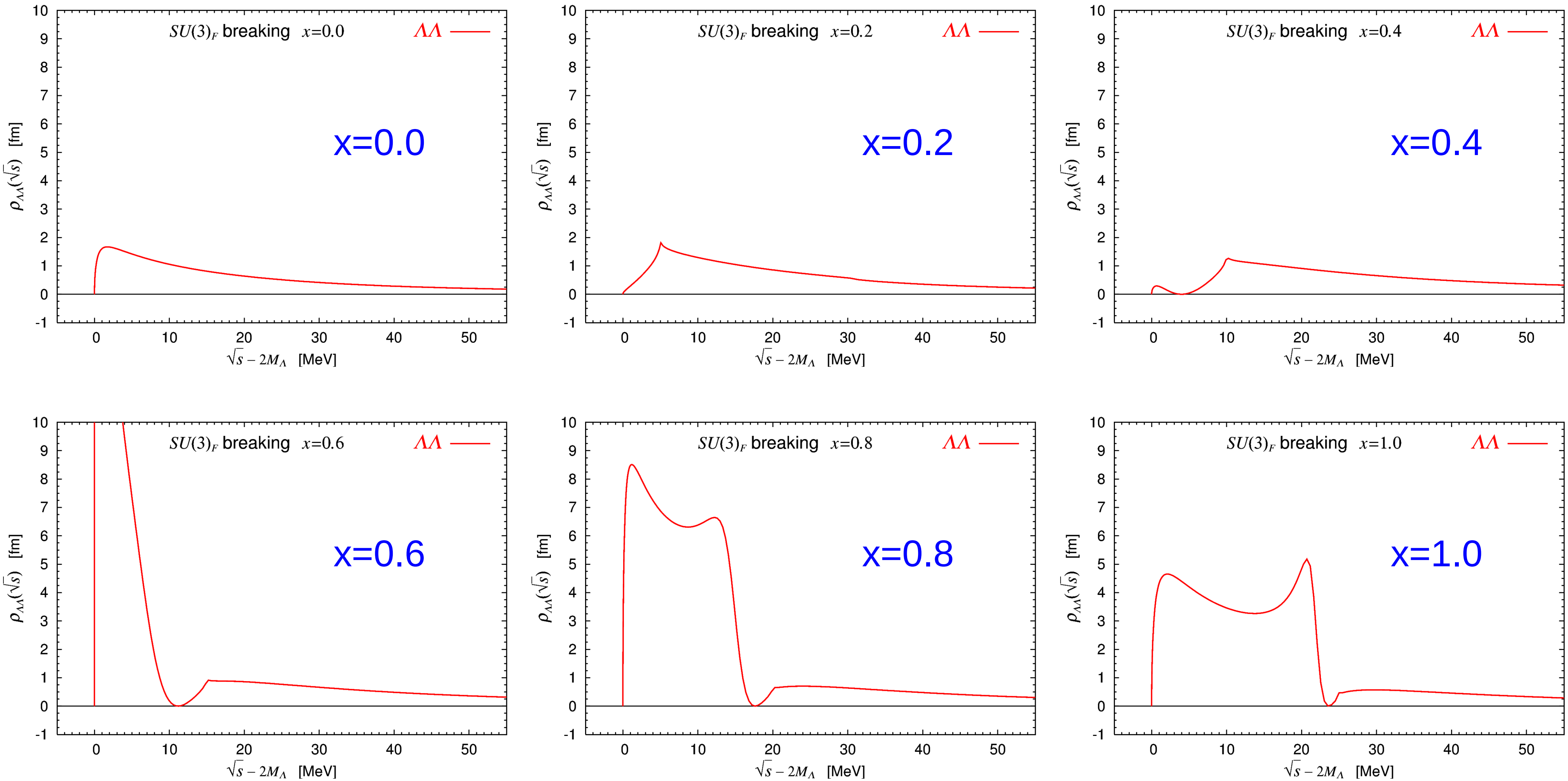
- H approaches the ΛΛ threshold from below and go though it.

# H-dibaryon in CSM




- Energy eigenvalues in the Complex-Scaling-Method.
- H comes 3 MeV below the NΞ threshold at the empirical SU(3)$_F$ breaking in this phenomenological trial calculation.

# ΛΛ invariant-mass-spectrum




- Invariant-mass-spectrum of ΛΛ calculated in S-wave dominance.

$$\rho_{\Lambda\Lambda}(\sqrt{s}) = |\,1 - S^{l=0}_{\Lambda\Lambda}\,|^2 \,/\, k$$

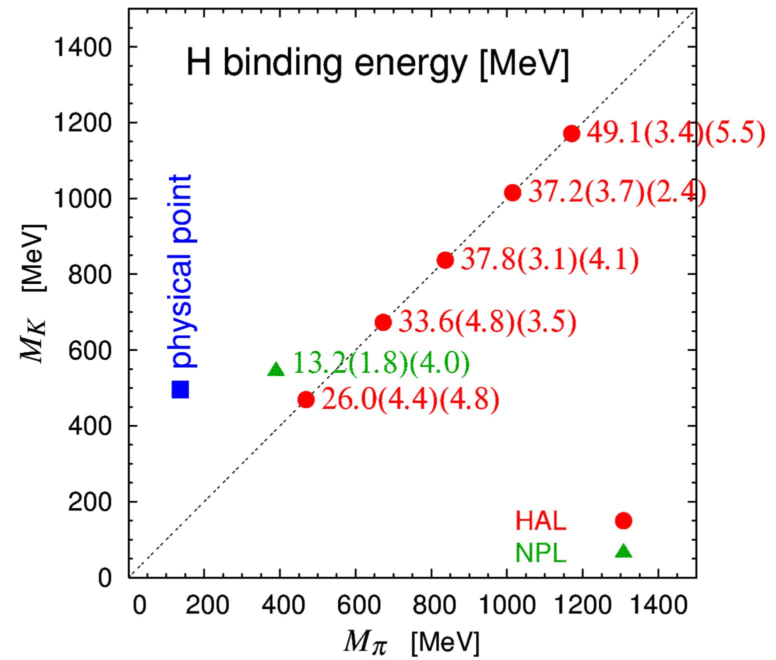
- We may have a chance to find H in experiments counting two Λ.

# Summary

- ✯ We've introduced motivation and purpose.
  - We want to explore for the H-dibaryon in QCD by using Lattice.
  - We start with flavor SU(3) limit to avoid complication.
- ✯ We've explained our approach.
  - We extract observables exclusively through the interaction potential.
  - By using both time and spacial derivative of the NBS w.f. at each point, we can extract the potential even without the ground-state-saturation.
- ✯ We've carried out full QCD lattice simulations for the H.
  - $32^3 \times 32$ lattice, L=3.87 [fm], Iwasaki gauge, clover quark
- ✯ We've found a bound=stable H-dibaryon
  - in flavor SU(3) symmetric world at $M_{PS}$ = 470 [MeV] – 1.17 [GeV],
  - its binding energy is 20 – 50 [MeV] depending on the quark mass,
- ✯ We've estimated H-dibaryon in the real world.
  - w/ phenomenological $SU(3)_F$ breaking, H may be a resonance.

# Summary & Outlook

- Plot of H binding energy from recent full QCD simulations.
  - SR. Beane etal [NPLQCD colla.] Phys. Rev. Lett. 106, 162001 (2011), arXive: 1109.2889[hep-lat].
  - Obtained binding energy from the two groups looks consistent.
  - But, all data points are still away from the physical point.

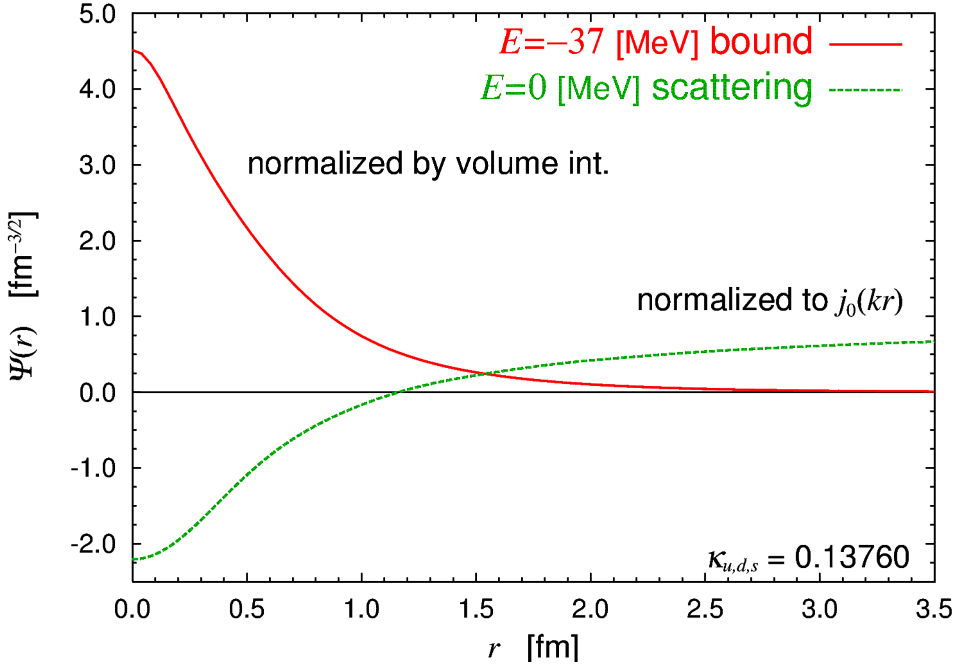


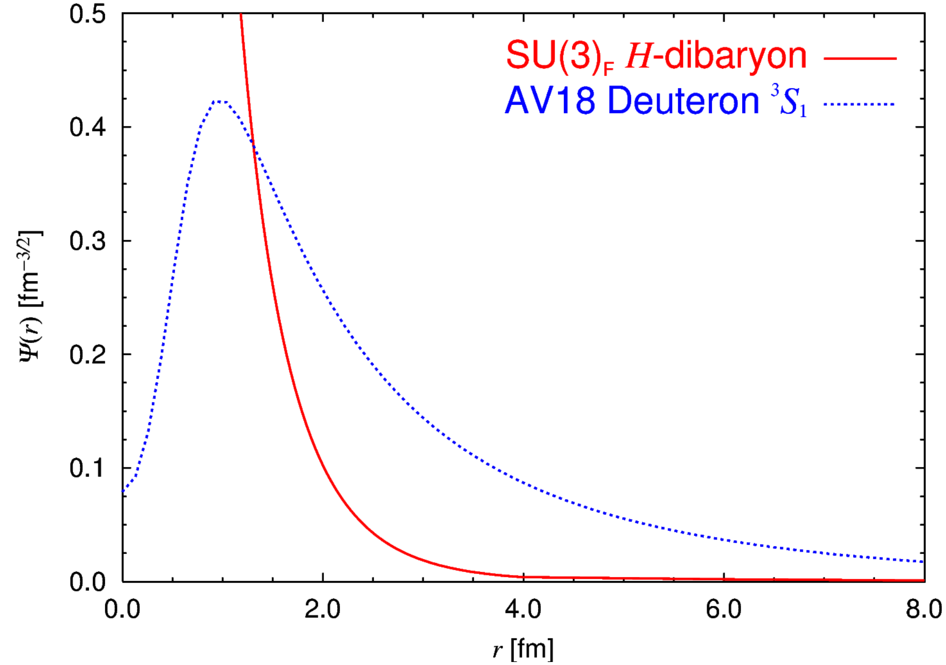

- We'll continue this study and put more data on this plot.
- We'll take flavor SU(3) breaking into account soon.
- We'll obtain information of H-dibaryon in the real world in near future.

# Thank You!

# Backup slides

# Size of H-dibaryon




- Left: Wave function of the lowest two states of $V^{(1)}$.
  - the measured NBS w.f. is a superposition of red and green with the finite volume effect.
- Right: Comparison between the H and the physical deuteron.
  - One can get feeling of the H-dibaryon. compact.

# FAQ

1. Does your potential depend on the choice of source?

2. Does your potential depend on choice of operator at sink?

3. Does your potential $U(r,r')$ or $V(r)$ depend on energy?

# FAQ

1. Does your potential depend on the choice of source?

➔ No. Some sources may enhance excited states in NBS w.f. However, the contamination is not problem in our new technique.

2. Does your potential depend on choice of operator at sink?

➔ Yes. It can be regarded as the “scheme” to define a potential. Note that a potential itself is not physical observable. We'll obtain unique result for observables irrespective to the choice, as long as the potential U(r,r') is deduced exactly.

3. Does your potential U(r,r') or V(r) depend on energy?

➔ By definition, U(r,r') is non-local but energy independent. While, determination and validity of its leading term V(r) obtained here, depend on energy because of the truncation. However, we know that the dependence in NN case is very small (thanks to our choice of sink operator = point) and negligible at least at $E_{lab.}$ = 0 – 90 MeV. We rely on this in the following. If we find the dependence, we'll determine the next leading term form it.

# FAQ

4. Do you think energy dependence of $V^{(1)}(r)$ is also small?

5. Is the H a compact six-quark object or a tight BB bound state?

# FAQ

4. Do you think energy dependence of $V^{(1)}(r)$ is also small?

➔ Yes. Because a large energy dependence means that

$$\left[2M_B - \frac{\nabla^2}{2\mu} + V_{\underline{Gr}}(\vec{r})\right]\phi_{Gr}(\vec{r})e^{-E_{Gr}t} = E_{Gr}\phi_{Gr}(\vec{r})e^{-E_{Gr}t}$$

$$\left[2M_B - \frac{\nabla^2}{2\mu} + V_{\underline{1\mathrm{st}}}(\vec{r})\right]\phi_{1\mathrm{st}}(\vec{r})e^{-E_{1\mathrm{st}}t} = E_{1\mathrm{st}}\phi_{1\mathrm{st}}(\vec{r})e^{-E_{1\mathrm{st}}t}$$

then $V(\vec{r}) \equiv \frac{1}{2\mu}\frac{\nabla^2\psi(\vec{r},t)}{\psi(\vec{r},t)} - \frac{\frac{\partial}{\partial t}\psi(\vec{r},t)}{\psi(\vec{r},t)} - 2M_B$ would have a large $t$-dep.

5. Is the H a compact six-quark object or a tight BB bound state?

➔ Both. There is no distinct separation between two, because baryon is nothing but a 3-quark in QCD. Imagine a compact 6-quark object in $(0S)^6$ configuration. This configuration can be rewritten in a form of $(0S)^3 \times (0S)^3 \times \mathrm{Exp}(-a\, r^2)$ with relative coordinate r. This shows that a compact six-quark object can have a baryonic component, which we measure in the NBS w.f. We've established existence of a stable QCD eigenstate which couples to BB state. We do NOT insist that another "H" doesn't exist which cannot couple to BB.

# FAQ

6. Do you insist such a deeply bound H exists in the real world?

7. What is the meaning of $\sqrt{\langle r^2 \rangle}$ of H?

# FAQ

6. Do you insist such a deeply bound H exists in the real world?

➔ No. With SU(3)F breaking, three BB thresholds in S=−2,I=0 sector split as $E^{\mathrm{Th}}_{\Lambda\Lambda} < E^{\mathrm{Th}}_{N\Xi} < E^{\mathrm{Th}}_{\Sigma\Sigma}$ . Therefore, we expect that the binding energy of H measured from $E^{\mathrm{Th}}_{\Lambda\Lambda}$ is much smaller than the present value, or even H is above $E^{\mathrm{Th}}_{\Lambda\Lambda}$ in the real world.

7. What is the meaning of $\sqrt{\langle r^2 \rangle}$ of H?

➔ It is a measure of spacial distribution of baryonic component in H. It corresponds to the “point matter root mean square distance” of deuteron ( 2 x 1.9 = 3.8 [fm]).

# Problem

- Free 2-body energy spectrum in finite volume w/ periodic B.C.

$$p_{x,y,z} = \frac{2 n_{x,y,z} \pi}{L} \quad \text{then} \quad K_{nx,ny,nz} = \left(n_x^2 + n_y^2 + n_z^2\right) \underline{\left(\frac{2\pi}{L}\right)^2 \frac{1}{2\mu}}$$

e.g. in $L = 2$ [fm], $2\mu = M = 1750$ [MeV] case, 220 [MeV]

therefor $E_{Gr} = 2M + 0,\ E_{1st} = 2M + 220,\ E_{2nd} = 2M + 440$

- Even with interaction, spectrum is essentially the same.
- State realized in lattice at time $t$ $\quad |t\rangle = |Gr\rangle + |1st\rangle \cdots$

NBS w.f. $\quad \psi(\vec{r}, t) = \phi_{Gr}(\vec{r}) e^{-E_{Gr} t} + \underline{\phi_{1st}(\vec{r}) e^{-E_{1st} t} \cdots}$

Depending on $\Delta E \equiv E_{1st} - E_{Gr}$, if $t$ is large enough

$\psi(\vec{r}, t) \simeq \phi_{Gr}(\vec{r}) e^{-E_{Gr} t}$ Ground State Saturation

Exponential tail

- On $L = 2$ [fm] lattice, G.S.S. is realized at $t \geq 10$ [a].

# SU(3) Octet Baryon operator

$$p_\alpha = \epsilon_{c_1 c_2 c_3} (C\gamma_5)_{d_1 d_2} \delta_{d_3 \alpha}\, u(\xi_1) d(\xi_2) u(\xi_3)$$
$$n_\alpha = \epsilon_{c_1 c_2 c_3} (C\gamma_5)_{d_1 d_2} \delta_{d_3 \alpha}\, u(\xi_1) d(\xi_2) d(\xi_3)$$

$$\Sigma_\alpha^+ = -\epsilon_{c_1 c_2 c_3} (C\gamma_5)_{d_1 d_2} \delta_{d_3 \alpha}\, u(\xi_1) s(\xi_2) u(\xi_3)$$
$$\Sigma_\alpha^0 = -\epsilon_{c_1 c_2 c_3} (C\gamma_5)_{d_1 d_2} \delta_{d_3 \alpha} \sqrt{\frac{1}{2}} \left[ d(\xi_1) s(\xi_2) u(\xi_3) + u(\xi_1) s(\xi_2) d(\xi_3) \right]$$
$$\Sigma_\alpha^- = -\epsilon_{c_1 c_2 c_3} (C\gamma_5)_{d_1 d_2} \delta_{d_3 \alpha}\, d(\xi_1) s(\xi_2) d(\xi_3)$$

$$\Lambda_\alpha = -\epsilon_{c_1 c_2 c_3} (C\gamma_5)_{d_1 d_2} \delta_{d_3 \alpha} \sqrt{\frac{1}{6}} \left[ d(\xi_1) s(\xi_2) u(\xi_3) + s(\xi_1) u(\xi_2) d(\xi_3) - 2u(\xi_1) d(\xi_2) s(\xi_3) \right]$$

$$\Xi_\alpha^0 = \epsilon_{c_1 c_2 c_3} (C\gamma_5)_{d_1 d_2} \delta_{d_3 \alpha}\, s(\xi_1) u(\xi_2) s(\xi_3)$$
$$\Xi_\alpha^- = \epsilon_{c_1 c_2 c_3} (C\gamma_5)_{d_1 d_2} \delta_{d_3 \alpha}\, s(\xi_1) d(\xi_2) s(\xi_3)$$

- With corrected phase $\quad \bar{1} = -\epsilon^{123} = -(ds - sd) = sd - ds$

# Irreducible BB source operator

$$\overline{BB^{(27)}} = +\sqrt{\frac{27}{40}}\,\overline{\Lambda}\,\overline{\Lambda} - \sqrt{\frac{1}{40}}\,\overline{\Sigma}\,\overline{\Sigma} + \sqrt{\frac{12}{40}}\,\overline{N}\,\overline{\Xi} \quad \text{or} \quad +\sqrt{\frac{1}{2}}\,\overline{p}\,\overline{n} + \sqrt{\frac{1}{2}}\,\overline{n}\,\overline{p}$$

$$\overline{BB^{(8s)}} = -\sqrt{\frac{1}{5}}\,\overline{\Lambda}\,\overline{\Lambda} - \sqrt{\frac{3}{5}}\,\overline{\Sigma}\,\overline{\Sigma} + \sqrt{\frac{1}{5}}\,\overline{N}\,\overline{\Xi}$$

$$\overline{BB^{(1)}} = -\sqrt{\frac{1}{8}}\,\overline{\Lambda}\,\overline{\Lambda} + \sqrt{\frac{3}{8}}\,\overline{\Sigma}\,\overline{\Sigma} + \sqrt{\frac{4}{8}}\,\overline{N}\,\overline{\Xi}$$

with

$$\overline{\Sigma}\,\overline{\Sigma} = +\sqrt{\frac{1}{3}}\,\overline{\Sigma^{+}}\,\overline{\Sigma^{-}} - \sqrt{\frac{1}{3}}\,\overline{\Sigma^{0}}\,\overline{\Sigma^{0}} + \sqrt{\frac{1}{3}}\,\overline{\Sigma^{-}}\,\overline{\Sigma^{+}}$$

$$\overline{N}\,\overline{\Xi} = +\sqrt{\frac{1}{4}}\,\overline{p}\,\overline{\Xi^{-}} + \sqrt{\frac{1}{4}}\,\overline{\Xi^{-}}\,\overline{p} - \sqrt{\frac{1}{4}}\,\overline{n}\,\overline{\Xi^{0}} - \sqrt{\frac{1}{4}}\,\overline{\Xi^{0}}\,\overline{n}$$

$$\overline{BB^{(10^*)}} = +\sqrt{\frac{1}{2}}\,\overline{p}\,\overline{n} - \sqrt{\frac{1}{2}}\,\overline{n}\,\overline{p}$$

$$\overline{BB^{(10)}} = +\sqrt{\frac{1}{2}}\,\overline{p}\,\overline{\Sigma^{+}} - \sqrt{\frac{1}{2}}\,\overline{\Sigma^{+}}\,\overline{p}$$

$$\overline{BB^{(8a)}} = +\sqrt{\frac{1}{4}}\,\overline{p}\,\overline{\Xi^{-}} - \sqrt{\frac{1}{4}}\,\overline{\Xi^{-}}\,\overline{p} - \sqrt{\frac{1}{4}}\,\overline{n}\,\overline{\Xi^{0}} + \sqrt{\frac{1}{4}}\,\overline{\Xi^{0}}\,\overline{n}$$

# Various Theoretical Approaches to Nuclei